A comparison with Rehbinder's THM solution

CSIRO

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1 Rehbinder's THM solution

1.1 Introduction

Rehbinder[1] derives analytical solutions of cylindrically-symmetric and spherically-symmetric THM problems in certain limits. The solution is a steady-state THM solution of a pressurised and heated cavity of radius r_{cavity} in a finite domain of radius r_{outer}. In the cylindrically-symmetric case the cavity is a cylinder of radius r_{cavity} and the domain is a cylinder of radius $r_{\mathrm{outer}} > r_{\mathrm{cavity}}$. In this document only the cylindrically-symmetric situation is explored. The cylindrical coordinates are denoted by (r, ϕ, z).

1.2 Initial and boundary conditions

The initial conditions are zero porepressure, $P_f(t = 0) = 0$; zero temperature, $T(t = 0) = 0$; and zero displacement $u(t = 0) = 0$.

The boundary conditions are at the cavity wall are

$$
\begin{aligned}
P_f(r_{\mathrm{cavity}}) &= P_0 \ , \\
T(r_{\mathrm{cavity}}) &= T_0 \ , \\
\sigma_{rr}^{\mathrm{eff}}(r_{\mathrm{cavity}}) &= 0 \ .
\end{aligned}
\tag{1.1}
$$

The latter condition implies the total radial stress is $-P_0$ at the cavity wall, corresponding to the fluid in the cavity pushing on the cavity wall. The boundary conditions at the outer radius are

$$
\begin{aligned}
P_f(r_{\mathrm{outer}}) &= 0 \ , \\
T(r_{\mathrm{outer}}) &= 0 \ ,
\end{aligned}
\tag{1.2}
$$

and either $\sigma_{rr}^{\mathrm{eff}}(r_{\mathrm{outer}}) = 0$ or $u(r_{\mathrm{outer}}) = 0$. In this document the former is called the "free outer" boundary condition, while the latter is called the "fixed outer" boundary condition.

1.3 Simplifying assumptions

In order to derive the solution, Rehbinder makes various simplifications. Translated to the language of the other PorousFlow documents these are as follows.

[1] G Rehbinder (1995) "Analytical solutions of stationary coupled thermo-hydro-mechanical solutions" Int J Rock Mech Min Sci and Geomech Abstr 32, 453–463

1. There is no gravity.
2. All quantities depend only on the radial coordinate, r, and not the angular coordinate ϕ or the axial coordinate z.
3. Plane-strain is assumed, so $\epsilon_{zz} = 0$.
4. There is only one fully-saturated fluid phase that contains one component.
5. The fluid relative permeability is unity.
6. The fluid density is constant. In the numerical simulation below, the density is assumed to obey $\rho = \rho_0 e^{P_f/K_f}$, with $\rho_0 = 1000\,\mathrm{kg.m^{-3}}$, and $K_f = 10^3\,\mathrm{GPa}$.
7. The fluid dynamic viscosity is constant. In the numerical simulation below, the dynamic viscosity is $\nu = 10^{-3}\,\mathrm{Pa.s}$.
8. The Biot coefficient is unity $\alpha_B = 1$.
9. The fluid internal energy is assumed to be linear in the temperature. In the numerical simulation below, $\mathcal{E} = CT$ where $C = 1000\,\mathrm{J.kg^{-1}.K^{-1}}$.
10. The fluid enthalpy is assumed to be equal to the fluid internal energy.
11. The velocity of the solid skeleton is zero: $\mathbf{v}_s = 0$. To model this using PorousFlow, the "`VolumetricExpansion`" Kernels are not included.
12. The rock-matrix density is constant. In the numerical simulation below, the value $\rho_R = 2500\,\mathrm{kg.m^3}$ is chosen.
13. The rock-matrix specific heat capacity is constant. In the numerical simulation below, the value $C_R = 1000.\mathrm{J.kg^{-1}.K^{-1}}$ is chosen.
14. The rock deforms elastically (so there is no plastic heating). In the numerical simulation below, the Young's modulus is $E = 10\,\mathrm{GPa}$ and the Poisson's ratio is $\nu = 0.2$.
15. The porosity is constant. In the simulation below $\phi = 0.1$ is chosen.
16. The permeability is constant. In the simulation below $k = 10^{-12}\,\mathrm{m}^2$ is chosen.
17. The thermal conductivity of the rock-fluid system is constant. In the simulation below $\lambda = 10^6\,\mathrm{J.s^{-1}.m^{-1}.K^{-1}}$ is chosen.
18. Steady-state heat flow, fluid-flow and mechanical deformation has been reached (Rehbinder equation (41)). Using the aove assumptions, this is true if $t > \rho_s c_s r_{\mathrm{cavity}}/\lambda = r_{\mathrm{cavity}}$ (the left-hand-side is measured in seconds, the right-hand-side in metres). In the simulation below, steady-state is achieved by not including any time-derivative Kernels.

19. The liquid flow is quasi-stationary, in the sense that the diffusion of heat is much slower than the diffusion of pore pressure (Rehbinder equation (32)). Using the above assumptions this may be written as $\phi\lambda\mu/K_f \ll \rho_s c_s k$ where λ is the thermal conductivity of the rock-fluid system, and $\rho_s c_s = (1-\phi)\rho_R C_R + \phi\rho C = 2.35\,\mathrm{MJ.K^{-1}.m^{-3}}$. Using the above values, this condition reads $10^6 \ll 2.35\times 10^{10}$ which is clearly satisfied.

20. The liquid flow is quasi-stationary, in the sense that a pressure change in the cavity is transmitted instantaneously through the pores to the outer boundary (Rehbinder equation (33)). Using the above assumptions this may be written as $r_{\mathrm{outer}} - r_{\mathrm{cavity}} \ll \sqrt{kK_f/(\phi\nu)} = 100\,\mathrm{m}$. In the simulation below, $r_{\mathrm{cavity}} = 0.1\,\mathrm{m}$ and $r_{\mathrm{outer}} = 1\,\mathrm{m}$ are chosen.

21. The heat is mainly conducted through the matrix, and a negligible part is convected with the flux. This is true if the Peclet number is very small (Rehbinder equation (35))
$$\mathrm{Pe} = \frac{\rho c \alpha_T T_0 E k}{\mu\lambda(1-\nu)} \ll 1 \ . \tag{1.3}$$
(Rehbinder writes this in terms of a reference temperature, T_{ref}, which is chosen here to be T_0.) Using the above values this is $\mathrm{Pe} = 1.25\times 10^{-5}\alpha_T T_0 \ll 1$. In the simulation below, $\alpha_T = 10^{-6}\,\mathrm{K}^{-1}$ and $T_0 = 1000\,\mathrm{K}$ are chosen, yielding $\mathrm{Re} = 0.0125$.

Rehbinder's derives the solution of this THM problem as an expansion in the Peclet number. I shall not write the solution here as it is fairly lengthy. The paper contains a few minor typos and they are corrected in the accompanying `thm_rehbinder.py` script.

2 Comparison with PorousFlow

The PorousFlow module is designed to simulate THM problems. The output compares favourably with Rehbinder's analytical solution, as demonstrated in the figures below.

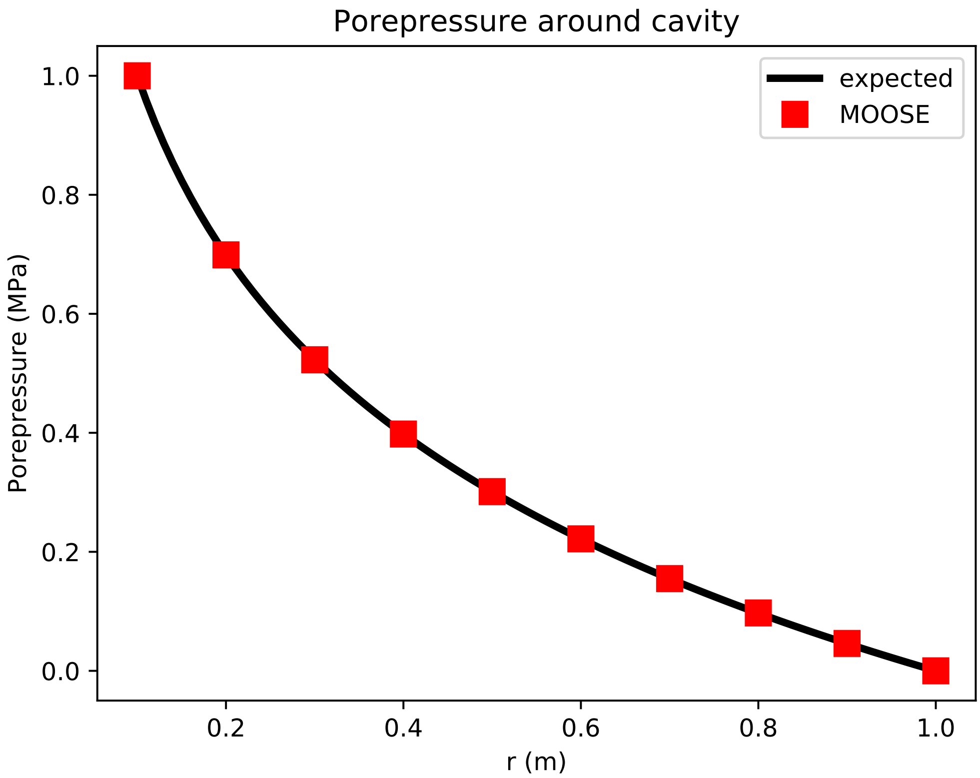

Figure 2.1: Comparison between the MOOSE result (squares) and the analytic expression derived by Rehbinder for the porepressure.

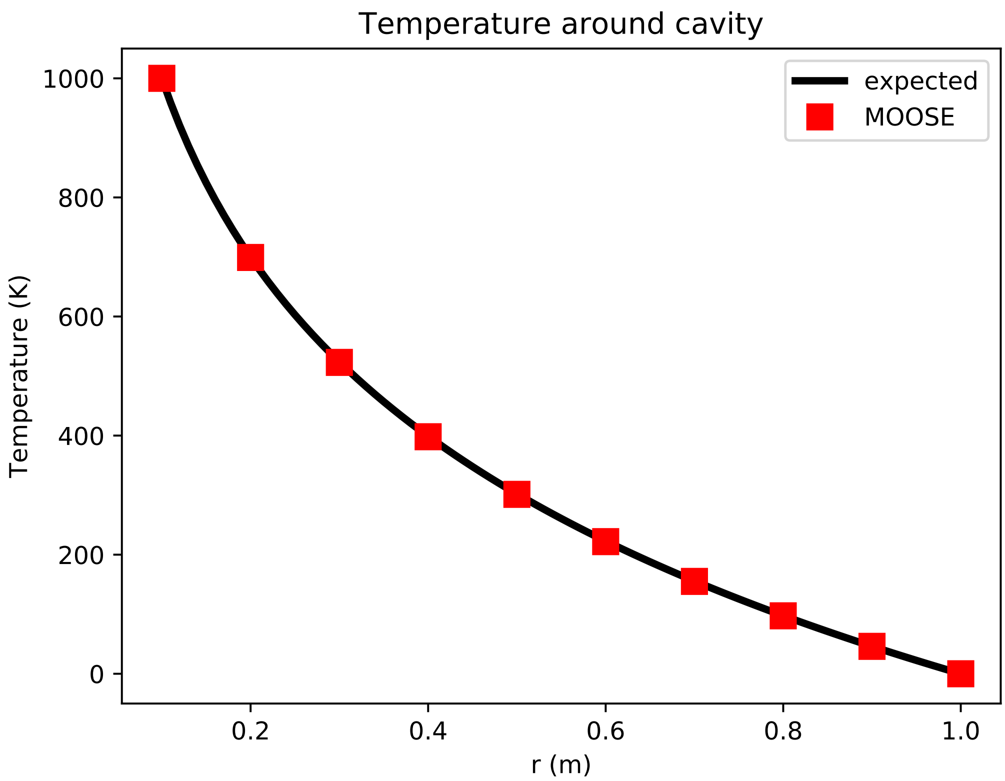

Figure 2.2: Comparison between the MOOSE result (squares) and the analytic expression derived by Rehbinder for the temperature.

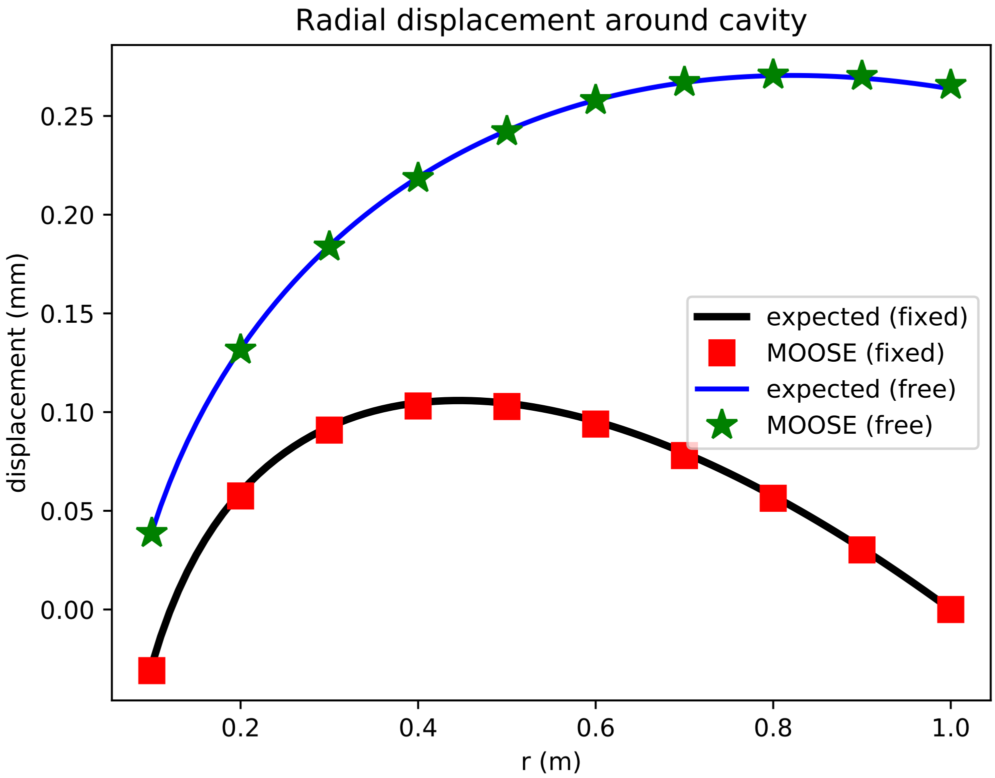

Figure 2.3: Comparison between the MOOSE results and the analytic expressions derived by Rehbinder for the radial displacement. Both the fixed and free outer boundary conditions are shown.